LOCAL AND INTEGRAL TURBULENT BOUNDARY LAYER CHARACTERISTICS ON A CONVEX SURFACE IN DIFFUSOR FLOW

A. A. Khalatov, A. A. Avramenko, M. M. Mitrakhovich, and V. G. Velichenko

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A semiempirical method is proposed for computing the local and integral characteristics of the turbulent boundary layer on a convex surface under conditions of a positive longitudinal pressure gradient. By using the method proposed, graphical dependences are obtained that characterize the influence of curvature on the form-parameter, the thickness of the momentum loss, the thickness of the viscous sublayer, and the relative flow coefficient.

The turbulent flow on a surface with longitudinal curvature (convex, concave) is encountered extensively in engineering. It occurs on turbine and compressor blades, in gas-dynamic laser elements, in Laval nozzles, around aircraft, rocket, ship, etc. surfaces.

P. Bradshaw first showed [1] that the longitudinal curvature exerts an effect on the stream hydrodynamics even for values of $\delta/R_W = 0.003$, and should certainly be taken into account in computing flow and heat transfer processes when $\delta/R_W > 0.003...0.005$. Taking account of the influence of the surface longitudinal curvature on the turbulent boundary layer characteristics should be accomplished in both the differential and integral equations and in expressions for the semiempirical closure hypotheses [2]. For small values of the ratio δ/R_W a correction to the longitudinal curvature is introduced only in the equations of the semiempirical hypothesis while the boundary layer equation is written exactly as for a plane stream.

Different modifications of the semiempirical hypotheses for a plane stream are used to determine the turbulent shear stresses in curvilinear boundary layers [2]. Most widespread is the relationship expressing the L. Prandtl hypothesis to conserve the circulation or the angular velocity under a random displacement of a turbulent mole

$$\tau = \rho l^2 \left(\frac{\partial u}{\partial r} \pm \frac{u}{r} \right)^2. \tag{1}$$

The plus sign here corresponds to the first, and the minus to the second hypothesis.

There is also a curvature correction in the expression for the mixing path length [2]. Utilized most often is the equation

$$l = l_0 (1 + \beta \, \mathrm{Ri})^{-1}, \tag{2}$$

in which the Richardson number characterizes the influence of the streamline curvature on the turbulent transfer (Ri > 0 for a convex wall and Ri < 0 for a concave). As is shown in [3], the parameter β is a function of the stream curvature δ^{**}/R_W and is determined by the equation

$$\beta = \begin{cases} 6 - 2.956\, \mathrm{th} \left[0.718 \left(\dfrac{\delta^{**}}{R_w} 10^3 - 1 \right) \right] & \text{for} \quad \dfrac{\delta^{**}}{R_w} = (1 \ldots 3) \cdot 10^{-3}, \\ 10 \left(1 + 1230 \dfrac{\delta^{**}}{R_w} \right)^{-0.706} & \text{for} \quad \dfrac{\delta^{**}}{R_w} = (3 \ldots 8) \cdot 10^{-3}. \end{cases} \tag{3}$$

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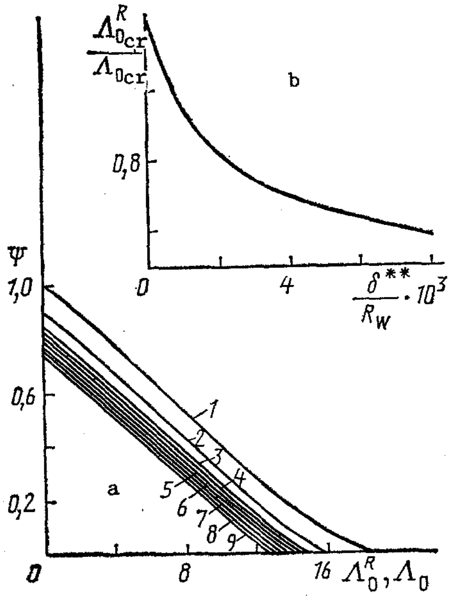

Fig. 1

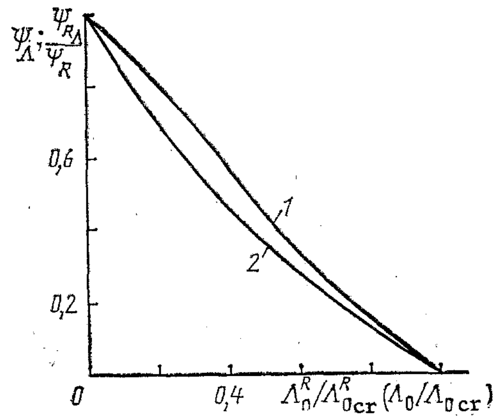

Fig. 2

Fig. 1. Influence of the longitudinal pressure gradient on the relative friction coefficient Ψ (a) (1-9 is the change in δ^{**}/R_w between 0 and $8\cdot10^{-3}$ with a $1\cdot10^{-3}$ spacing) and of the curvature on the critical value of the pressure gradient parameter (b).

Fig. 2. Influence of the relative longitudinal pressure gradient on the relative friction coefficient Ψ: 1) results of the present computation; 2) a flat surface [5].

Gradient-free flow around convex and concave surfaces is examined in [1, 3] etc. It is shown there that the surface friction coefficient for $\delta/R_w = 0.01$ is 10% less than the value on a flat surface and already 20% less for $\delta/R_w = 0.02$. The influence of the convex curvature on the local and integral characteristics of a turbulent boundary layer and also the friction law are studied in [3]. The influence of convex curvature on boundary layer characteristics under conditions of a positive pressure gradient, that is encountered extensively in engineering, is studied in this paper. The hypothesis of conservation of the velocity circulation (1) and also (2) and (3) are used for the analysis.

Transforming (1), we obtain the relative friction law

$$\Psi^{0.5} = \frac{(1-\bar{\Gamma}_1)(1+\delta/R_w)}{(c_{f_0}/2)^{0.5}\int\limits_{\xi_1}^{1}\left(1+\xi\frac{\delta}{R_w}\right)\frac{\bar{\tau}^{0.5}}{\bar{l}}\,d\xi}. \tag{4}$$

The influence of curvature on the relative friction coefficient in this equation is taken into account by the ratio δ/R_w, the expression (2) for ℓ, and also the tangential friction stress distribution over the turbulent boundary layer section.

To integrate (4) we compile a system of additional equations. We obtain the parameters on the sublayer boundary after solving the motion equation in the viscous sublayer domain

$$\omega^{(1)} = c_{f_0}/2\,(\mathrm{Re}^{**}/\bar{\delta}^{**})\left[\Psi\xi^{(1)} + \frac{\Lambda_0^R}{2}\xi^{(1)^2}\right], \tag{5}$$

$$\dot{R} = c_{f_0}/2\,(\mathrm{Re}^{**}/\delta^{**})^2(\Psi\xi^{(1)^2} + \Lambda_0^R\xi^{(1)^3}). \tag{6}$$

We use an expression [4] that agrees well with test data

$$\bar{\tau} = \bar{\tau}_0\Phi\xi\exp(\Phi\xi)/\mathrm{sh}(\Phi\xi)\left[1 + m\frac{\Phi\xi}{1+\Phi\xi}\right], \tag{7}$$

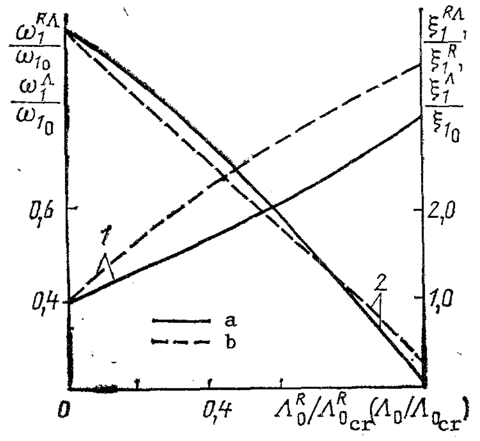

Fig. 3

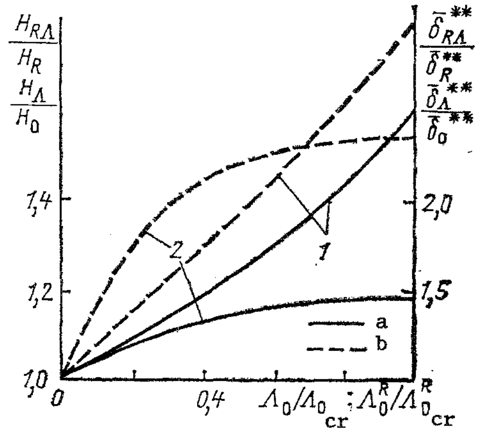

Fig. 4

Fig. 3. Influence of the relative longitudinal pressure gradient on the viscous sublayer thickness (1) and velocity on the viscous sublayer boundary (2): a) results of the present paper; b) flat surface [5].

Fig. 4. Influence of the relative longitudinal pressure gradient on the form-factor H (1) and momentum loss thickness $\bar{\delta}^{**}$ (2): a, b) see Fig. 3.

where

$$m = -0{,}8\left[1 + \exp\left(-\frac{E}{1 + 0{,}2E}\right)\exp(2|\Phi|\xi)\right];\ E = (|\Phi|\xi)^{0,5}\exp(0{,}2|\Phi|\xi);$$

$$\Phi = (\partial\bar{\tau}/\partial\xi)_w = \Lambda_0^R/\Psi - 2\bar{\delta}^{**-1}\delta^{**}/R_w,$$

for the tangential friction stress distribution over the turbulent boundary layer section under positive pressure gradient conditions.

The stream parameters on a flat plate (the standard boundary layer) are determined by the following equations

$$c_{f_0} = 2\,[5.89\,\lg(4.075\,Re^{**})]^{-2}, \tag{8}$$

$$\bar{\tau}_0 = 1 - 2.1\xi^{1,1} + 1.1\xi^{2,1}, \tag{9}$$

$$\bar{l}_0 = 0.14 - 0.08\,(1-\xi)^2 - 0.06\,(1-\xi)^4. \tag{10}$$

The system (1)-(10) describes a turbulent boundary layer on a curvilinear surface ($R_w \lesseqgtr 0$) in the presence of a longitudinal pressure gradient ($\Lambda_0{}^R \lesseqgtr 0$).

A program for computation on the electronic computer ES-1061 is compiled for the solution of this system of equations. Computations of the local and integral characteristics of a turbulent boundary layer are executed for a convex surface with a positive pressure gradient as the number Re** changes from $3\cdot10^3$ to 10^4 and the curvature parameter δ^{**}/R_w from $1\cdot10^{-3}$ to $8\cdot10^{-3}$. The computation was performed by successive approximation with previously assigned accuracy. The computation time for one modification did not exceed one minute. The quantity of partitions for calculation of the integral in (4) was increased until their influence on the absolute value of the results obtained ceased. Satisfactory accuracy was achieved for a number of partitions equal to 900.

Results of a computation of the relative friction coefficient are presented in Fig. 1a for different values of the pressure gradient parameter. It is seen from the figure that as the curvature grows, for all values of $\Lambda_0{}^R$ the magnitude of the relative function Ψ diminishes as compared with a flat surface. This is due to the stabilizing action of the convex wall on the turbulent boundary layer structure. The values of Ψ for $\Lambda_0 = 0$ correspond to the influence of positive curvature on the friction coefficient [3].

Results are represented in Fig. 1b of determining the critical value of the pressure gradient parameter ${\Lambda_0}_{cr}^R$ corresponding to stream separation from the surface ($\Psi = 0$). It is seen that the convex wall curvature results in earlier stream separation as compared with the flow on a flat surface (${\Lambda_0}_{cr}$). The value of the critical pressure gradient parameter diminishes 30% on a convex wall in the range of δ^{**}/R_w variation between 0 and $8 \cdot 10^{-3}$.

The results presented in Fig. 1 are represented in Fig. 2 in conformity with the approach examined in [5]. It follows from the figure that the results of computations for a curvilinear stream are described by a single dependence (line 1) for different values of the ratio δ^{**}/R_w and differ by not more than 20% from the line 2 corresponding to the flow with a positive pressure gradient on a flat surface. This means that taking account of the longitudinal surface curvature in the friction coefficient for a convex surface in a first approximation can be done because of correction of the critical pressure gradient parameter in the expression for the friction law for a curvilinear stream.

The influence of the positive pressure gradient on the local and integral turbulent boundary layer parameters on convex and flat surfaces is shown in Figs. 3 and 4. Values of the parameters on a curvilinear surface for gradient-free flow are taken from data in [3].

As follows from Fig. 3, upon approaching the separation point on a curvilinear surface, the positive pressure gradient increases the viscous boundary layer thickness less intensively than on a flat surface. The influence of the positive pressure gradient on the value of the velocity on the viscous sublayer boundary is practically identical for both rectilinear and curvilinear flows (Fig. 3). The surface curvature results in a less noticeable growth of the form-factor H and the relative momentum-loss thickness $\bar{\delta}^{**}$ under the action of a positive pressure gradient (Fig. 4).

According to the data of the present investigation, the critical parameters in the section of turbulent boundary layer separation equal $(\bar{\delta}_{\Lambda R}^{**}/\bar{\delta}_R^{**})_{cr} = 1.47$, $(H_{\Lambda R}/H_R) = 1.6$.

The computation method considered can, in principle, be used also to determine the turbulent boundary layer characteristics on a concave surface. To do this, the value of β should be used for this surface and in all the equations presented above to yield $R_w < 0$.

The dependence obtained for the relative friction coefficient can be utilized to close the integral boundary layer equation during its computation on a convex surface under positive pressure gradient conditions.

NOTATION

τ, τ_w, tangential stress in the stream and at the wall; $\bar{\tau} = \tau/\tau_w$; ρ, density; ℓ, mixing path length; u, longitudinal velocity component; $\Gamma = ur$, velocity circulation; y, distance from the wall; R_w, wall radius of curvature; $r = y + R_w$; $\Psi = (c_f/c_{f_0})_{Re^{**}}$, relative friction coefficient; c_f, friction coefficient on a convex surface in the presence of a pressure gradient; c_{f_0}, friction coefficient on a flat wall ($\Lambda_0 = 0$); Re^{**}, Reynolds number comprised from the momentum loss thickness; δ^{**}, momentum loss thickness; δ, boundary layer thickness; $\bar{\delta}^{**} = \delta^{**}/\delta$; $\xi = y/\delta$; $\bar{\ell} = \ell/\delta$; $\bar{\Gamma} = ur/[u_\infty(\delta + R_w)]$, dimensionless circulation velocity; $\omega = u/u_\infty$; $R = (y\nu^{-1}\partial u/\partial y)_{y=y_1}$, viscous sublayer stability parameter; $\Lambda_0 = \delta \tau_{w_0}{}^{-1} dp/dx$, pressure gradient parameter; $Ri = 2(\delta/R_w)\ell_0/(c_{f_0}/2\Psi\bar{\tau})^{0.5}$, Richardson number; H, form-factor; ν, kinematic viscosity. The subscripts: 0, flat surface; w, wall; 1, viscous sublayer; ∞, outer boundary of the boundary layer; R, curvature; Λ, pressure gradient, and cr, separation point.

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